Applied Statistics for Life Sciences

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Module 5: Statistical Modelling, Regression and ANOVA

Contents

The Least Squares

- A simple data set consists of data pairs (x_i, y_i), $i = 1, \ldots, n$, where x_i is called *independent variable* and y_i is called *dependent variable*
- We are looking for the model function of the form $y = a + bx$ such that it gives "best" fit to the data
- "best" in what sense?

Residuals

- A **residual** r_i is defined as the difference between the values of the dependent variable and the predicted values from the estimated model

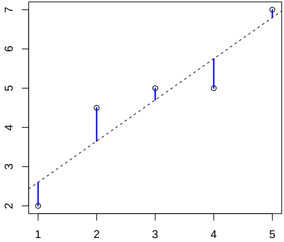
$$r_i = y_i - \hat{y}_i, \text{ where } \hat{y}_i = a + bx_i$$

- The least squares method defines "best" model as when

$$S = \sum_{i=1}^{n} r_i^2$$

is at minimum

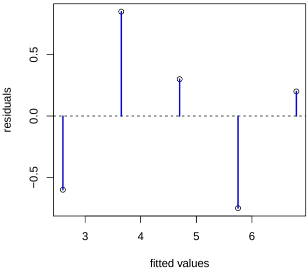
Regression Line



- Residuals are shown in blue
- Residuals are positive for data points above the line
- Residuals are negative for data points below the line
- Sum of squares of residuals is at minimum

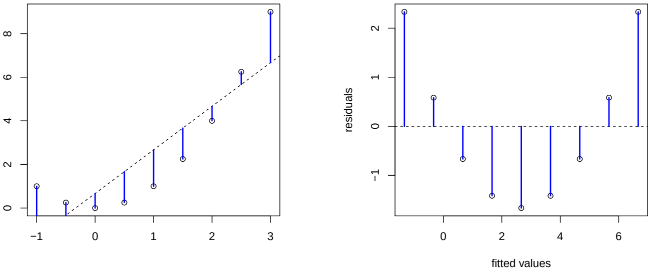
Residual plot

The residual plot is the scatterplot of residuals vs. fitted values, i.e., $y_i - \hat{y}_i \sim \hat{y}_i$



- The sum of the residuals w.r.t least square line is equal to zero

Residual plot



A pattern in the residual plot indicates that a non-linear model should be used

Influential Scores and Outliers

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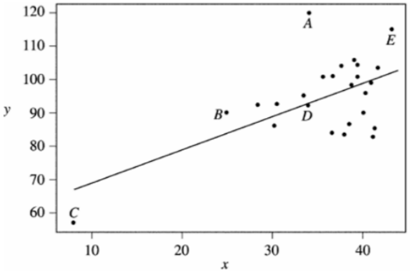
- In regression, an **outlier** is a data point with large residual
- An **influential score** is the data point which significantly influences the regression line
- If an influential score is removed from the sample, the regression line will change significantly

Problem 1.1

Which of the five points is an outlier, and which is an influential score?



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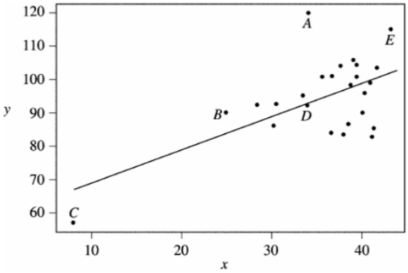


Solution

Correct: (A) is an outlier; (C) is an influential score

Solving the Regression

$$S = \sum_{i=1}^{n} r_i^2 = \sum_{i=1}^{n} (y_i - (a + bx_i))^2 \to \min$$

$$\begin{cases} \frac{\partial S}{\partial a} = -2\sum_{i=1}^{n}(y_i - (a + bx_i)) = 0 \\ \frac{\partial S}{\partial b} = -2\sum_{i=1}^{n} x_i(y_i - (a + bx_i)) = 0 \end{cases}$$

$$\begin{cases} \sum_{i=1}^{n} y_i = an + b\sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i y_i = a\sum_{i=1}^{n} x_i + b\sum_{i=1}^{n} x_i^2 \end{cases}$$

If $\sum_{i=1}^{n} x_i = 0$ then $b = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2}$. In general, $b = \frac{\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})}{\sum_{i=1}^{n}(x_i - \overline{x})^2}$

Regression Slope and Intercept

$$b = \frac{\sum(x_i - \overline{x})(y_i - \overline{y})}{\sum(x_i - \overline{x})^2} = \frac{s_{xy}}{s_{xx}},$$

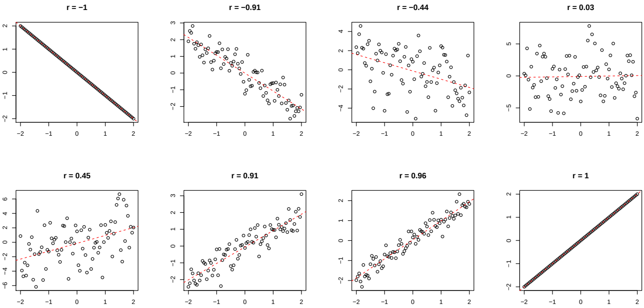
where

$$s_{xy} = \frac{1}{n-1}\sum(x_i - \overline{x})(y_i - \overline{y}),$$

$$s_{xx} = \frac{1}{n-1}\sum(x_i - \overline{x})(x_i - \overline{x}),$$

$$a = \overline{y} - b\overline{x}$$

Pearson Correlation Coefficient



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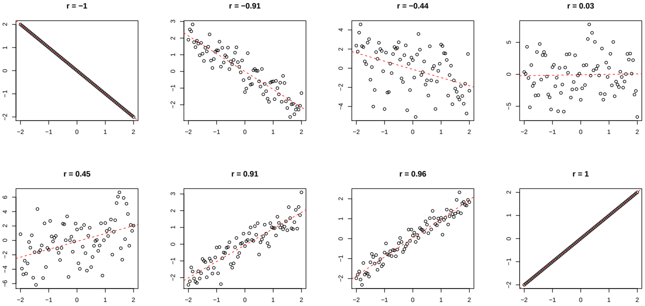

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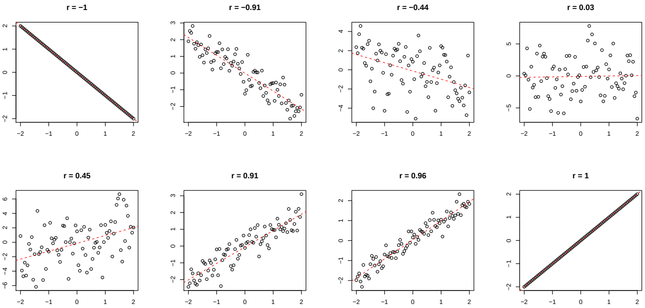

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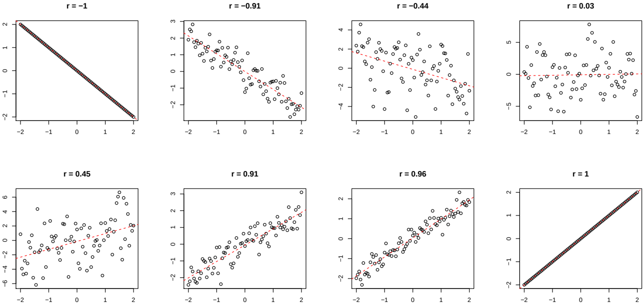

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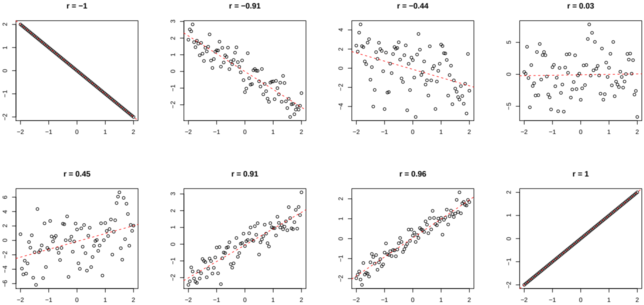

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- $r \in [-1, 1]$
- r and the regression slope have the same sign
- Regression slope is *not* determined by the value of r
- Variables with zero correlation are *uncorrelated* but not necessarily independent

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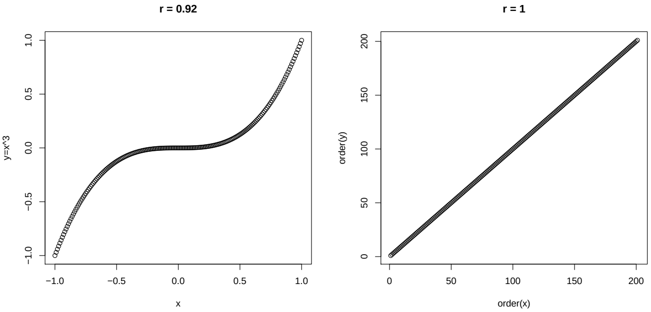

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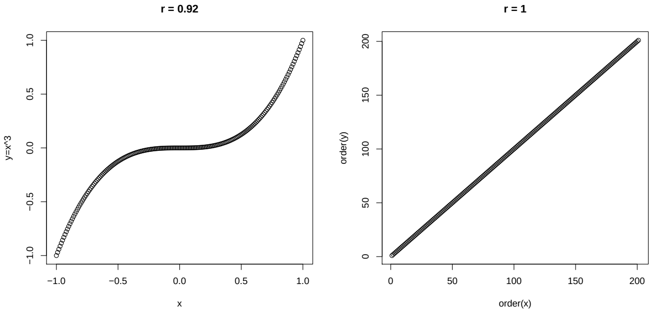

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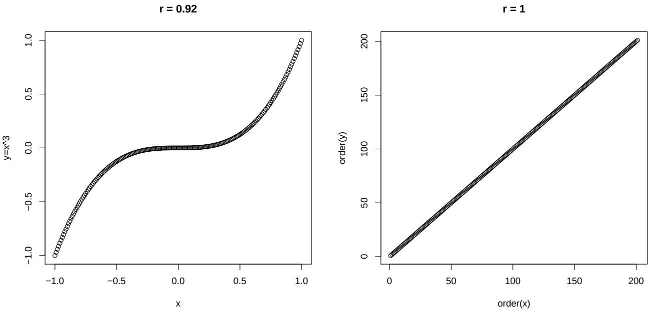

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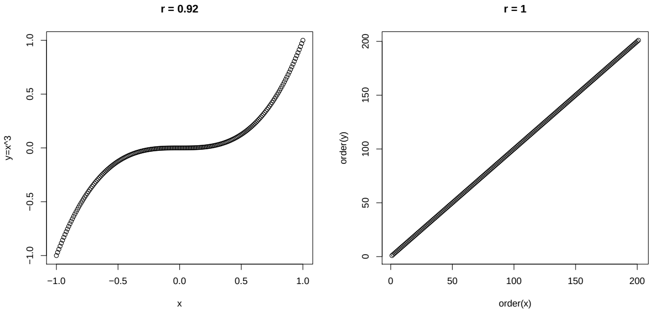

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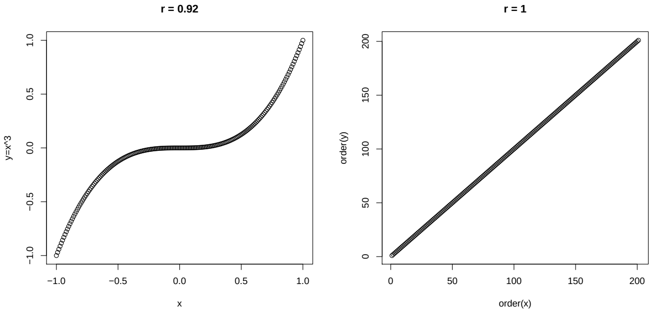

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- $r_s \in [-1, 1]$
- r_s is sensitive to the order of observations, not their absolute value
- r_s indicates the degree of *monotonous*, not necessarily linear dependence
- Unlike Pearson correlation coefficient, r_s is not sensitive to outliers or influential scores

Correlation and Regression Slope

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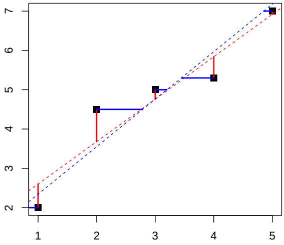
- $b = \frac{s_{xy}}{s_x^2} = r\frac{s_y}{s_x}$

Regression y vs. x and x vs. y



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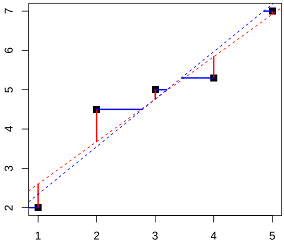

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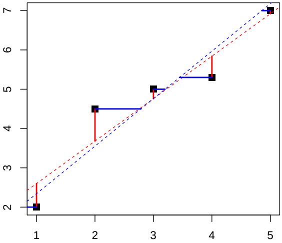

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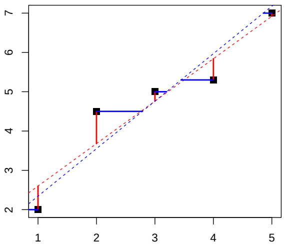

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- $b = \frac{s_{xy}}{s_x^2} = r\frac{s_y}{s_x}, b^* = \frac{s_{xy}}{s_y^2} = r\frac{s_x}{s_y}$
- $b \cdot b^* = r^2$, i.e., the product of slopes of inverse regression lines is $r^2 \leq 1$

Coefficient of Determination

$$R^2 = r^2 = \frac{SSX}{SST}$$

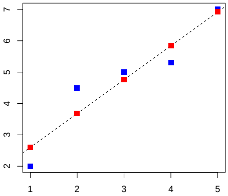
- SST = total sum of squares
- SSX = sum of squares explained by X
- SSE = sum of squares of residuals
- SST = SSX+SSE

- The square of the sample correlation coefficient, which is also known as the *coefficient of determination*, is the fraction of the variance in y that is accounted for by a linear fit of x.

Decomposition of Sums of Squares

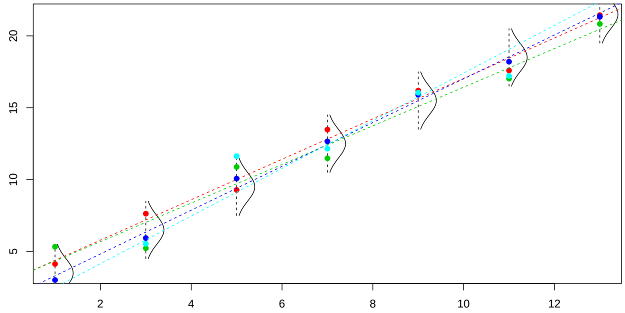
$$(n-1)s_Y^2 = \sum(y_i - \overline{y})^2 = \sum\left((y_i - \hat{y}_i) + (\hat{y}_i - \overline{y})\right)^2 =$$
$$\sum(y_i - \hat{y}_i)^2 + \sum(\hat{y}_i - \overline{y})^2 + 2 \cdot \text{cross-product} = SSE + SSX$$

$$\text{cross-product} = \sum(y_i - \hat{y}_i)(\hat{y}_i - \overline{y}) = \sum r_i(a + bx_i - a - b\overline{x}) = b\sum r_i(x_i - \overline{x}) = 0$$



y_i vs. $\hat{y}_i$

From Least Squares to Statistics



- The regression line is a result of random sampling
- Different samples produce different lines
- There is a family of lines for given population; you see just one

Linear Regression Model

The model postulates that $y_i = \alpha + \beta x_i + e_i$, where

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  - e_i and e_j are independent for $i \neq j$

SE of the Regression Slope

- $\hat{\alpha} = a$ and $\hat{\beta} = b$ from LS are unbiased effective estimators of α and β
- $\text{SE}(\hat{\beta}) = \frac{\sigma_e}{\sqrt{\sum(x_i - \overline{x})^2}}$
- $\hat{\sigma}_e = \sqrt{\frac{\sum(y_i - \hat{y}_i)^2}{n-2}} = \sqrt{\frac{SSE}{n-2}} = \sqrt{MSE}$

Confidence Interval for the Regression Slope

- $\beta = \hat{\beta} \pm t_{\alpha/2}(df) SE(\hat{\beta})$
- $df = n - 2$
- $SE(\hat{\beta}) = \frac{\sigma_e}{\sqrt{\sum(x_i - \overline{x})^2}} = \frac{\sigma_e}{s_x\sqrt{n-1}}$

Problem 1.2

Growth hormones are often used to increase the weight gain in chickens. In an experiment using 15 chickens, five different doses of growth hormone were injected into chickens (three for each dose) and the subsequent weight gain was recorded. An experimenter plots the data and finds that a linear relationship appears to hold. The output of the software is

Term	Estimate	Std Error	t-ratio	P
Intercept	4.5458533	0.616518	7.37	0.0001
dose	4.83233426	1.016403	4.75	0.0004

- *What is the equation for the fitted line?*
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- $$H_0 : \beta = 0$$
  $$H_a : \beta > 0$$
  $t = \frac{4.83-0}{1.016} = 4.75$, $\mathrm{P}(t(13) > 4.75) = 0.0002$, *i.e.,* H_0 *is rejected at the 1% significance level. That is, the weight gain in chicken significantly depends on the dose of the growth hormone*

Problem 1.3

A marine biologist wants to test the effect of water temperature on the average dive duration for sea otters. Seven otters are available for the study. The biologist collects the data with the following summary statistics. $\sum X = 80$, $\sum Y = 639$, $\sum X^2 = 1088$, $\sum Y^2 = 60457$, $\sum XY = 7888$. *Find the regression line and a 95% confidence interval for the regression slope.*

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- $\overline{X} = \frac{80}{7} = 11.4$, $\overline{Y} = \frac{639}{7} = 91.3$, $s_x^2 = \frac{1088 - 7 \cdot 11.4^2}{6} = 29.7$, $s_y^2 = \frac{60457 - 7 \cdot 91.3^2}{6} = 351.2$, $s_{xy} = \frac{7888 - 7 \cdot 11.4 \cdot 91.3}{6} = 100.4$

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- $\texttt{duration} = 52.77 + 3.38\texttt{temp}$
- $r = \frac{s_{xy}}{\sqrt{s_x^2 s_y^2}} = \frac{100.4}{\sqrt{29.7 * 351.2}} = 0.98$

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- $b = \frac{s_{xy}}{s_X^2} = \frac{100.4}{29.7} = 3.38$, $a = \overline{Y} - b\overline{X} = 91.3 - 3.38 * 11.4 = 52.77$
- $\texttt{duration} = 52.77 + 3.38\texttt{temp}$
- $r = \frac{s_{xy}}{\sqrt{s_x^2 s_y^2}} = \frac{100.4}{\sqrt{29.7 * 351.2}} = 0.98$
- $SSY = (n-1)s_y^2 = 6 * 351.2$, $SSE = (1 - R^2)SSY = (1 - 0.98^2) * 6 * 351.2 = 83.44$, $SE(\hat{\beta}) = \frac{\sqrt{83.44/(7-2)}}{\sqrt{6 * 29.7}} = 0.30$

Problem 1.3

A marine biologist wants to test the effect of water temperature on the average dive duration for sea otters. Seven otters are available for the study. The biologist collects the data with the following summary statistics. $\sum X = 80$, $\sum Y = 639$, $\sum X^2 = 1088$, $\sum Y^2 = 60457$, $\sum XY = 7888$. *Find the regression line and a 95% confidence interval for the regression slope.*

Solution

- $\overline{X} = \frac{80}{7} = 11.4$, $\overline{Y} = \frac{639}{7} = 91.3$, $s_x^2 = \frac{1088-7\cdot 11.4^2}{6} = 29.7$, $s_y^2 = \frac{60457-7\cdot 91.3^2}{6} = 351.2$, $s_{xy} = \frac{7888-7\cdot 11.4\cdot 91.3}{6} = 100.4$
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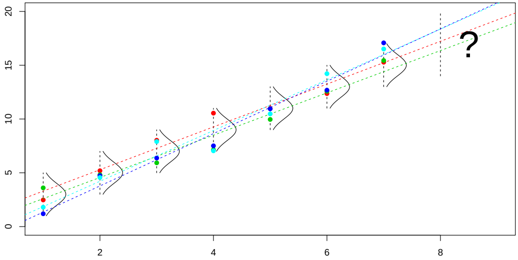
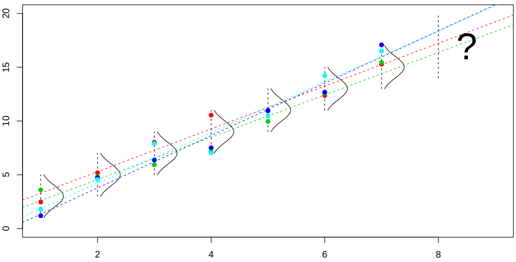
- $r = \frac{s_{xy}}{\sqrt{s_x^2 s_y^2}} = \frac{100.4}{\sqrt{29.7 * 351.2}} = 0.98$
- $SSY = (n-1)s_y^2 = 6 * 351.2$, $SSE = (1-R^2)SSY = (1-0.98^2) * 6 * 351.2 = 83.44$, $SE(\hat{\beta}) = \frac{\sqrt{83.44/(7-2)}}{\sqrt{6*29.7}} = 0.30$
- $\beta = \hat{\beta} \pm t_{0.025}(5)SE(\hat{\beta}) = 3.38 \pm 2.57 * 0.30 = 3.38 \pm 0.77$, *i.e., we are 95% confident that the dive duration increases by on average* 3.38 ± 0.77 *minutes with each additional Celcius degree of water*

Confidence vs. Prediction Interval



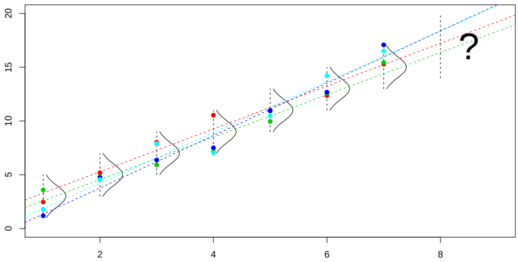
- Suppose I fuel my car 7 days a week, from Sunday to Sunday, each day at a randomly chosen gas station. I get a sample of gasoline prices for 7 days.

Confidence vs. Prediction Interval



- Suppose I fuel my car 7 days a week, from Sunday to Sunday, each day at a randomly chosen gas station. I get a sample of gasoline prices for 7 days.
- *Confidence interval* is for the average gasoline price on Monday

Confidence vs. Prediction Interval



- Suppose I fuel my car 7 days a week, from Sunday to Sunday, each day at a randomly chosen gas station. I get a sample of gasoline prices for 7 days.
- *Confidence interval* is for the average gasoline price on Monday
- *Prediction interval* is for a gasoline price at a randomly chosen gas station on Monday

Confidence vs. Prediction Interval

- x_0 is the new point
- Confidence Interval

$$\hat{y}_0 = \alpha + \beta x_0 = a + bx_0 \pm t_{\alpha/2}(n-2)\hat{\sigma}_e\sqrt{\frac{1}{n} + \frac{(x_0 - \overline{x})^2}{\sum(x_i - \overline{x})^2}}$$

- Prediction Interval

$$\hat{y}_0 = \alpha + \beta x_0 = a + bx_0 \pm t_{\alpha_2}(n-2)\hat{\sigma}_e\sqrt{\frac{1}{n} + \frac{(x_0 - \overline{x})^2}{\sum(x_i - \overline{x})^2} + 1}$$

Problem 1.4 (problem 1.3 revisited)

A marine biologist wants to test the effect of water temperature on the average dive duration for sea otters. Seven otters are available for the study. The biologist collects the data with the following summary statistics. $\sum X = 80$, $\sum Y = 639$, $\sum X^2 = 1088$, $\sum Y^2 = 60457$, $\sum XY = 7888$. *Find the 95% confidence and prediction intervals for the dive duration at 25 degrees Celsius.*

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Solution

- $\overline{X} = \frac{80}{7} = 11.4$, $\overline{Y} = \frac{639}{7} = 91.3$, $s_x^2 = \frac{1088 - 7 \cdot 11.4^2}{6} = 29.7$, $s_y^2 = \frac{60457 - 7 \cdot 91.3^2}{6} = 351.2$, $s_{xy} = \frac{7888 - 7 \cdot 11.4 \cdot 91.3}{6} = 100.4$

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Solution

- $\overline{X} = \frac{80}{7} = 11.4$, $\overline{Y} = \frac{639}{7} = 91.3$, $s_x^2 = \frac{1088 - 7 \cdot 11.4^2}{6} = 29.7$, $s_y^2 = \frac{60457 - 7 \cdot 91.3^2}{6} = 351.2$, $s_{xy} = \frac{7888 - 7 \cdot 11.4 \cdot 91.3}{6} = 100.4$
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A marine biologist wants to test the effect of water temperature on the average dive duration for sea otters. Seven otters are available for the study. The biologist collects the data with the following summary statistics. $\sum X = 80$, $\sum Y = 639$, $\sum X^2 = 1088$, $\sum Y^2 = 60457$, $\sum XY = 7888$. *Find the 95% confidence and prediction intervals for the dive duration at 25 degrees Celsius.*

Solution

- $\overline{X} = \frac{80}{7} = 11.4$, $\overline{Y} = \frac{639}{7} = 91.3$, $s_x^2 = \frac{1088 - 7 \cdot 11.4^2}{6} = 29.7$, $s_y^2 = \frac{60457 - 7 \cdot 91.3^2}{6} = 351.2$, $s_{xy} = \frac{7888 - 7 \cdot 11.4 \cdot 91.3}{6} = 100.4$
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- $\hat{\sigma} = \sqrt{MSE} = \sqrt{83.44/(7-2)} = 4.8$, $t_{0.025}(5) = 2.57$

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A marine biologist wants to test the effect of water temperature on the average dive duration for sea otters. Seven otters are available for the study. The biologist collects the data with the following summary statistics. $\sum X = 80$, $\sum Y = 639$, $\sum X^2 = 1088$, $\sum Y^2 = 60457$, $\sum XY = 7888$. *Find the 95% confidence and prediction intervals for the dive duration at 25 degrees Celsius.*

Solution

- $\overline{X} = \frac{80}{7} = 11.4$, $\overline{Y} = \frac{639}{7} = 91.3$, $s_x^2 = \frac{1088 - 7 \cdot 11.4^2}{6} = 29.7$, $s_y^2 = \frac{60457 - 7 \cdot 91.3^2}{6} = 351.2$, $s_{xy} = \frac{7888 - 7 \cdot 11.4 \cdot 91.3}{6} = 100.4$
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- $\hat{\sigma} = \sqrt{MSE} = \sqrt{83.44/(7-2)} = 4.8$, $t_{0.025}(5) = 2.57$
- *Confidence* $\hat{y}_0 = 52.77 + 3.38 \cdot 25 \pm 2.57 \cdot 4.8\sqrt{\frac{1}{7} + \frac{(25 - 11.4)^2}{6 * 29.7^2}} = 137.3 \pm 5.2$ *min, i.e., we are 95% confident that the average dive duration at* $25^o C$ *is* 137.3 ± 5.2 *minutes.*

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A marine biologist wants to test the effect of water temperature on the average dive duration for sea otters. Seven otters are available for the study. The biologist collects the data with the following summary statistics. $\sum X = 80$, $\sum Y = 639$, $\sum X^2 = 1088$, $\sum Y^2 = 60457$, $\sum XY = 7888$. *Find the 95% confidence and prediction intervals for the dive duration at 25 degrees Celsius.*

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- $\overline{X} = \frac{80}{7} = 11.4$, $\overline{Y} = \frac{639}{7} = 91.3$, $s_x^2 = \frac{1088 - 7 \cdot 11.4^2}{6} = 29.7$, $s_y^2 = \frac{60457 - 7 \cdot 91.3^2}{6} = 351.2$, $s_{xy} = \frac{7888 - 7 \cdot 11.4 \cdot 91.3}{6} = 100.4$
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- *Prediction* $\hat{y}_0 = 52.77 + 3.38 \cdot 25 \pm 2.57 \cdot 4.8\sqrt{\frac{1}{7} + \frac{(25-11.4)^2}{6*29.7^2} + 1} = 137.3 \pm 13.4$ *min, i.e., we are 95% confident that when a sea otter dives at 25$^o C$ next time, the duration will be* 137.3 ± 13.4 *minutes*

ANOVA: Analysis of Variance

- A collection of models, in which the variance of the observed set is partitioned into components due to explanatory variables
- Assumptions:
  - Independence of observations
  - The distributions in each of the groups are normal
  - Variance homogeneity, called homoscedasticity: the variance of data in groups should be the same.

ANOVA: Analysis of Variance

A manager wishes to determine whether the mean times required to complete a certain task differ for the three levels of employee training. He randomly selected 10 employees with each of the three levels of training.

Level	n	$\overline{x}$	s^2
Advanced	10	24.2	21.54
Intermediate	10	27.1	18.64
Beginner	10	30.2	17.76

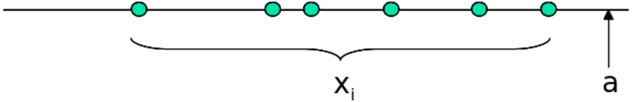
Do the data provide sufficient evidence to indicate that the mean times required to complete a certain task differ for at least two of the three levels of training?

One-way ANOVA example

Three different milling machines were being considered for purchase by a manufacturer. Potentially, the company would be purchasing hundreds of these machines, so it wanted to make sure it made the best decision. Initially, five of each machine were borrowed, and each was randomly assigned to one of 15 technicians (all technicians were similar in skill). Each machine was put through a series of tasks and rated using a standardized test. The higher the score on the test, the better the performance of the machine. The data are:

Machine 1	Machine 2	Machine 3
24.50	28.40	26.10
23.50	34.20	28.30
26.40	29.50	24.30
27.10	32.20	26.20
29.90	30.10	27.80

Steiner's Theorem



$$I(x_1, x_2, \ldots, x_n; a) = \sum_{i=1}^{n} (x_i - a)^2 = \text{Moment of inertia}$$

$$I(x_1, x_2, \ldots, x_n; a) = I(x_1, x_2, \ldots, x_n; \overline{x}) + n(\overline{x} - a)^2$$

Decomposition of Sum of Squares

- SST = SSA + SSE
- SST = total sum of squares
- SSA = sum of squares for factor A
- SSE = sum of squares of errors

Decomposition of Sum of Squares

Observed

	M1	M2	M3	mean
	24.50	28.40	26.10	
	23.50	34.20	28.30	
	26.40	29.50	24.30	
	27.10	32.20	26.20	
	29.90	30.10	27.80	
Mean	26.28	30.88	26.54	27.90

Expected

	M1	M2	M3	mean
	26.28	30.88	26.54	
	26.28	30.88	26.54	
	26.28	30.88	26.54	
	26.28	30.88	26.54	
	26.28	30.88	26.54	
Mean	26.28	30.88	26.54	27.90

$$(24.50 - 27.90)^2 + (23.50 - 27.90)^2 + \cdots + (29.90 - 27.90)^2 + \cdots =$$

$$(24.50 - 26.28)^2 + (23.50 - 26.28)^2 + \cdots + (29.90 - 26.28)^2 + 5 * (26.28 - 27.90)^2 \cdots = SSE + SSA$$

$$SSE = \sum_{i=1}^{n} \sum_{j=1}^{m} (x_{ij} - \overline{x}_{\bullet j})^2$$

$$SSA = n \sum_{j=1}^{m} (\overline{x}_{\bullet j} - \overline{x}_{\bullet\bullet})^2$$

$$SSE = SST - SSA$$

Decomposition of Sum of Squares

	1	2	3	
	x_{11}	x_{12}	x_{13}	
	x_{21}	x_{22}	x_{23}	
	x_{31}	x_{32}	x_{33}	
	x_{41}	x_{42}	x_{43}	
	x_{51}	x_{52}	x_{53}	
mean	$x_{\bullet 1}$	$x_{\bullet 2}$	$x_{\bullet 3}$	$x_{\bullet\bullet}$

$$SST = \sum_{i=1}^{n}\sum_{j=1}^{m}(x_{ij} - \overline{x}_{\bullet\bullet})^2 = \sum_{j=1}^{m}\left(\sum_{i=1}^{n}(x_{ij} - \overline{x}_{\bullet j})^2 + n(\overline{x}_{\bullet j} - \overline{x}_{\bullet\bullet})^2\right) =$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m}(x_{ij} - \overline{x}_{\bullet j})^2 + n\sum_{j=1}^{m}(\overline{x}_{\bullet j} - \overline{x}_{\bullet\bullet})^2 = SSE + SSA$$

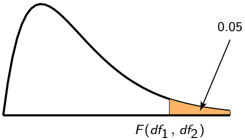
Assumption: $x_{ij} - \overline{x}_{\bullet j} \sim \mathcal{N}(0, \sigma^2)$ **are independent***

One-way ANOVA table

- SST = SSA + SSE = Total sum of squares
- SSA = Sum of squares Factor A
- SSE = Sum of squares Error
- MSA = Mean sum of squares Factor
- MSE = Mean sum of squares Error

	SS	df	MS	F	P-value
Factor	SSA	k-1	SSA/(k-1)	MSA/MSE	$P(F > \ldots)$
Error	SSE	n-k	SSE/(N-k)		
Total	SST	n-1			

Fisher F-distribution



df_1	$df_1 = 2$	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	161.45	18.51	10.13	7.71	6.61	5.99	5.59	5.32	5.12	4.96
2	199.50	19.00	9.55	6.94	5.79	5.14	4.74	4.46	4.26	4.10
3	215.71	19.16	9.28	6.59	5.41	4.76	4.35	4.07	3.86	3.71
4	224.58	19.25	9.12	6.39	5.19	4.53	4.12	3.84	3.63	3.48
5	230.16	19.30	9.01	6.26	5.05	4.39	3.97	3.69	3.48	3.33
6	233.99	19.33	8.94	6.16	4.95	4.28	3.87	3.58	3.37	3.22
7	236.77	19.35	8.89	6.09	4.88	4.21	3.79	3.50	3.29	3.14
8	238.88	19.37	8.85	6.04	4.82	4.15	3.73	3.44	3.23	3.07
9	240.54	19.38	8.81	6.00	4.77	4.10	3.68	3.39	3.18	3.02
10	241.88	19.40	8.79	5.96	4.74	4.06	3.64	3.35	3.14	2.98

$$\mathrm{P}\left(F(df_1, df_2) < x\right) = \mathrm{P}\left(\frac{1}{F(df_1, df_2)} > \frac{1}{x}\right) = \mathrm{P}\left(F(df_2, df_1) > \frac{1}{x}\right)$$

Solution to the example

Source of Variation	SS	df	MS	F	P-value
Between Groups	66.77	2	33.39	7.14	0.009073
Within Groups	56.13	12	4.68		
Total	122.9	14			

Conclusion: H_0 is rejected at 5% significance level, i.e., there is enough evidence to suspect that machines are different.

Problem 2.1

Some varieties of nematodes feed on the roots of lawn grasses and crops such as strawberries and tomatoes. Four brands of nematocides are to be compared. Twelve plots of land of comparable fertility that were suffering from nematodes were planted with a crop. The yields of each plot were recorded and part of the ANOVA table appears below:

Source of Variation	SS	df	MS	F	P-value
Nematocides	3.456	*	*	*	*
Error	1.200	8	*		
Total	4.656	11			

Find the value of F and P-value.

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Source of Variation	SS	df	MS	F	P-value
Nematocides	3.456	*	*	*	*
Error	1.200	8	*		
Total	4.656	11			

Find the value of F and P-value.

Solution

Source of Var	SS	df	MS	F	P-value
Nematocides	3.456	11-8=3	$\frac{3.4456}{3} = 1.152$	$\frac{1.152}{0.15} = 7.68$	$\text{P}(F(3,8) > 7.68) = 0.009$
Error	1.200	8	$\frac{1.2}{8} = 0.15$		
Total	4.656	11			

Two-way ANOVA

- **One-way ANOVA** Group A is given vodka, Group B is given gin, and Group C is given a placebo. Groups are tested with a memory task.
- **Two-way ANOVA** In an experiment testing the effects of expectations, subjects are randomly assigned to four groups:
  - expect vodka — receive vodka
  - expect vodka — receive placebo
  - expect placebo — receive vodka
  - expect placebo — receive placebo

  Each group is then tested on a memory task.

Decomposition of Sum of Squares

- SST = SSA + SSB + SSE
- SST = total sum of squares
- SSA = sum of squares for factor A
- SSB = sum of squares for factor B
- SSE = sum of squares of errors

Decomposition of Sum of Squares

				mean
	x_{11}	x_{12}	x_{13}	$x_{1\bullet}$
	x_{21}	x_{22}	x_{23}	$x_{2\bullet}$
	x_{31}	x_{32}	x_{33}	$x_{3\bullet}$
	x_{41}	x_{42}	x_{43}	$x_{4\bullet}$
	x_{51}	x_{52}	x_{53}	$x_{5\bullet}$
mean	$x_{\bullet 1}$	$x_{\bullet 2}$	$x_{\bullet 3}$	$x_{\bullet\bullet}$

$$SST = \sum_{i=1}^{n}\sum_{j=1}^{m}(x_{ij} - \overline{x}_{\bullet\bullet})^2 = \sum_i\sum_j(x_{ij} - \overline{x}_{i\bullet})^2 + m\sum_i(\overline{x}_{i\bullet} - \overline{x}_{\bullet\bullet})^2 =$$

$$\sum_i\sum_j\left((x_{ij} - \overline{x}_{i\bullet} - \overline{x}_{\bullet j} + \overline{x}_{\bullet\bullet}) + (\overline{x}_{\bullet j} - \overline{x}_{\bullet\bullet})\right)^2 + SSA = \sum_i\sum_j(x_{ij} - \overline{x}_{i\bullet} - \overline{x}_{\bullet j} + \overline{x}_{\bullet\bullet})^2 +$$

$$n\sum_j(\overline{x}_{\bullet j} - \overline{x}_{\bullet\bullet})^2 + SSA = \sum_i\sum_j(x_{ij} - \overline{x}_{i\bullet} - \overline{x}_{\bullet j} + \overline{x}_{\bullet\bullet})^2 + SSB + SSA = SSA + SSB + SSE$$

Assumption: $x_{ij} - \overline{x}_{i\bullet} - \overline{x}_{\bullet j} + \overline{x}_{\bullet\bullet} \sim \mathcal{N}(0, \sigma^2)$ **are independent***

Two-way ANOVA example

Three different milling machines were being considered for purchase by a manufacturer. Potentially, the company would be purchasing hundreds of these machines, so it wanted to make sure it made the best decision. Initially, five of each machine were borrowed. *Machines are operated by 5 different crew technicians:*

	Machine 1	Machine 2	Machine 3
Crew 1	24.50	28.40	26.10
Crew 2	23.50	34.20	28.30
Crew 3	26.40	29.50	24.30
Crew 4	27.10	32.20	26.20
Crew 5	29.90	30.10	27.80

What is the Error Term?

Observed

	M1	M2	M3	mean
Crew 1	24.50	28.40	26.10	26.30
Crew 2	23.50	34.20	28.30	28.70
Crew 3	26.40	29.50	24.30	26.70
Crew 4	27.10	32.20	26.20	28.50
Crew 5	29.90	30.10	27.80	29.30
mean	26.28	30.88	26.54	27.90

Expected

	M1	M2	M3	mean
Crew 1	24.70	29.30	25.00	26.30
Crew 2	27.00	31.60	27.30	28.70
Crew 3	25.10	29.70	25.40	26.70
Crew 4	26.90	31.50	27.10	28.50
Crew 5	27.60	32.20	27.90	29.30
mean	26.28	30.88	26.54	27.90

$$X_{ij} = 24.50, \quad \overline{x}_{i\bullet} + \overline{x}_{\bullet j} - \overline{x}_{\bullet\bullet} = 26.28 + 26.30 - 27.90 = 24.70$$

Two-way ANOVA Table

	SS	df	MS	F	P-value
Factor A	SSA	a-1	SSA/(a-1)	MSA/MSE	$P(F > \ldots)$
Factor B	SSB	b-1	SSB/(b-1)	MSB/MSE	$P(F > \ldots)$
Error	SSE	n-a-b+1	SSE/(N-a-b+1)		
Total	SST	n-1			

Source of Variation	SS	df	MS	F	P-value
Rows	19.89	4	4.97	1.098	0.4199
Columns	66.77	2	33.39	7.37	0.0153
Error	36.23	8	4.53		
Total	122.9	14			

Conclusion: At 5% significance level there is enough evidence to suspect that machines are different, but not enough evidence to suspect that operators are different.

Decomposition of Sum of Squares

				mean
	$\{x_{11},\dots\}$	$\{x_{12},\dots\}$	$\{x_{13},\dots\}$	$x_{1\bullet}$
	$\{x_{21},\dots\}$	$\{x_{22},\dots\}$	$\{x_{23},\dots\}$	$x_{2\bullet}$
	$\{x_{31},\dots\}$	$\{x_{32},\dots\}$	$\{x_{33},\dots\}$	$x_{3\bullet}$
	$\{x_{41},\dots\}$	$\{x_{42},\dots\}$	$\{x_{43},\dots\}$	$x_{4\bullet}$
	$\{x_{51},\dots\}$	$\{x_{52},\dots\}$	$\{x_{53},\dots\}$	$x_{5\bullet}$
mean	$x_{\bullet 1}$	$x_{\bullet 2}$	$x_{\bullet 3}$	$x_{\bullet\bullet}$

$$SST = \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{\alpha=1}^{k}(x_{ij,\alpha} - \overline{x}_{\bullet\bullet})^2 = \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{\alpha=1}^{k}(x_{ij,\alpha} - \overline{x}_{ij,\bullet})^2 + k\sum_{i=1}^{n}\sum_{j=1}^{m}(x_{ij,\bullet} - \overline{x}_{\bullet\bullet})^2 =$$
$$SSE + SSA + SSB + \sum_{i}\sum_{j}\sum_{\alpha}(x_{ij,\alpha} - \overline{x}_{ij,\alpha})^2 = SSE + SSA + SSB + SSAB$$

SSAB = interaction of factors A and B

Assumption: SSE is the sum of squares of independent $\mathcal{N}(0,\sigma^2)$

Problem 2.2

The following data on corn yields are obtained by planting three seed types using five fertilizers.

	Fert I	Fert II	Fert III	Fert IV	Fert V
Seed A-402	106, 110	95, 100	94, 107	103, 104	100, 102
Seed B-894	110, 112	98, 99	100, 101	108, 112	105, 107
Seed C-952	94, 97	86, 87	98, 99	99, 101	94, 98

Test at 5% significance level the hypothesis that corn yield depends on the seed type, fertilizer type, or the combination of the two.

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Solution

By using R statistics `summary(aov(value seed+fert+seed*fert, data))`

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
seed	2	512.87	256.43	28.28	0.0000
fert	4	449.47	112.37	12.39	0.0001
seed:fert	8	143.13	17.89	1.97	0.1221
Residuals	15	136.00	9.07		

Given

	Fert I	Fert II	Fert III	Fert IV	Fert V
Seed A-402	106, 110	95, 100	94, 107	103, 104	100, 102
Seed B-894	110, 112	98, 99	100, 101	108, 112	105, 107
Seed C-952	94, 97	86, 87	98, 99	99, 101	94, 98

$SS = 1241.467$

By cell

	Fert I	Fert II	Fert III	Fert IV	Fert V	mean
Seed A-402	108.00	97.50	100.50	103.50	101.00	102.10
Seed B-894	111.00	98.50	100.50	110.00	106.00	105.20
Seed C-952	95.50	86.50	98.50	100.00	96.00	95.30
mean	104.83	94.17	99.83	104.50	101.00	

$SS = 552.7$

By row (seed)

	Fert I	Fert II	Fert III	Fert IV	Fert V
Seed A-402	102.10	102.10	102.10	102.10	102.10
Seed B-894	105.20	105.20	105.20	105.20	105.20
Seed C-952	95.30	95.30	95.30	95.30	95.30

$SS = 256.4$

By column (fert)

	Fert I	Fert II	Fert III	Fert IV	Fert V
Seed A-402	104.83	94.17	99.83	104.50	101.00
Seed B-894	104.83	94.17	99.83	104.50	101.00
Seed C-952	104.83	94.17	99.83	104.50	101.00

$SS = 224.7$

ANOVA

	Df	**Sum Sq**	**Mean Sq**	**F value**	**Pr(>F)**
seed	**2**	**256.4*2=512.8**	**256.43**	**28.28**	**0.0000**
fert	**4**	**224.7*2=449.4**	**112.37**	**12.39**	**0.0001**
seed:fert	**8**	**(552.7-256.4-224.7)*2=143**	**17.89**	**1.97**	**0.1221**
Residuals	**15**	**1241.5-552.7*2=136.1**	**9.07**		
Toal	**29**				

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- Two-way ANOVA deals with two factors and, possibly, their interactions